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Coulomb screening in mesoscopic noise: a kinetic approach

Frederick Green† and Mukunda P Das‡

† GaAs IC Prototyping Facility, CSIRO Telecommunications and Industrial Physics, PO Box 76, Epping NSW 1710, Australia
‡ Department of Theoretical Physics, Research School of Physical Sciences and Engineering, The Australian National University, Canberra ACT 0200, Australia

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Abstract. Coulomb screening, together with degeneracy, is characteristic of the metallic electron gas. While there is little trace of its effects in transport and noise in the bulk, at mesoscopic scales the electronic fluctuations start to show appreciable Coulomb correlations. Within completely standard Boltzmann and Fermi-liquid frameworks, we analyse these phenomena and their relation to the mesoscopic fluctuation-dissipation theorem, which we prove. We identify two distinct screening mechanisms for mesoscopic fluctuations. One is the self-consistent response of the contact potential in a non-uniform system. The other couples to scattering, and is an exclusively non-equilibrium process. Contact-potential effects renormalize all thermal fluctuations, at all scales. Collisional effects are relatively short ranged and modify non-equilibrium noise. We discuss ways to detect these differences experimentally.

(Some figures in this article are in colour only in the electronic version; see `www.iop.org`)

1. Introduction

In a previous paper [1] (GD; see also reference [2]) we proposed a semi-classical kinetic theory of carrier fluctuations in metallic conductors down to mesoscopic dimensions. Our microscopic description is based on two canonical frameworks: the Boltzmann transport equation and the theory of charged Fermi liquids. Boltzmann kinetics is needed to compute non-equilibrium carrier distributions and fluctuations, given their equilibrium properties as input. It is Fermi-liquid theory, *and that theory alone*, that can supply the necessary microscopic form of the input.

Any inhomogeneity in a conductor of mesoscopic size begins to show itself even if the device is strongly metallic. For such systems the microscopic origins of non-equilibrium Coulomb screening have not been examined, at least within orthodox kinetic theory. The immediate goal of this work is to provide such a description. Its scope should cover not just low-field phenomena, but also the technologically important high-field regime.

The formalism of GD treats the degenerate electron gas independently of the self-consistent Coulomb fields which accompany, and modify, fluctuations of electric charge. This purely Fermi-liquid analysis is related conceptually to the Lindhard approximation [3]; with it, one can derive a quantitative connection between non-equilibrium fluctuations of the current and the rate of dissipation of electrical energy (Joule heating). In the bulk weak-field limit this is nothing but the linear fluctuation-dissipation theorem; at high fields, the connection expresses the scaling of excess thermal noise with the ambient thermodynamic temperature. For degenerate carriers, such scaling is an unavoidable consequence of the asymptotic boundary

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conditions for the open environment to which the conductor is connected: local equilibrium, and local charge neutrality.

There are other semi-classical treatments of Coulomb screening for mesoscopic noise in metals [4,5]. They rely mainly on Langevin stochastics [6] and drift-diffusion phenomenology [7], avoiding substantive engagement with Fermi-liquid physics; notably, the conserving sum rules [3]. The shortcomings of diffusive noise theory are discussed elsewhere [8, 9]. By contrast, a well-defined numerical method for Coulomb suppression has been developed by González and others [10] to study non-degenerate shot noise; their approach adheres to standard Boltzmann kinetics. We leave our own study of shot noise to a future paper. For a preliminary account, see reference [2].

In the present work, our most significant result is to show that the fluctuations of the dissipated power are functionally distinct from the underlying microscopic fluctuations of the current. Quite simply, each behaves differently from the other. The consequences of this separation will be seen only in a conductor that is (a) metallic and (b) non-uniform. Its experimental significance, and the corroboration of our model, are most likely to be realized in a two-dimensional quantum channel designed to meet conditions (a) and (b).

If it is corroborated, our kinetic description will offer a unique probe of mesoscopic phenomena that have not been considered to date. Moreover, these effects can be analysed equally well in the regime of *non-linear* transport and noise. That is the import of our work. The fact that it treats, simultaneously, the disparate issues of non-uniform Coulomb screening, degeneracy and non-linearity requires an extended technical commitment.

A prime example of a system with strong in-built screening effects is the two-dimensional electron gas, self-consistently confined in the quantum well of a III–V heterojunction [11]; we examine how to set up a first-principles kinetic description for these significant systems, as well as others. In section 2 we revisit the kinetic formalism of GD and the structure of its Green functions. The analysis is then extended, in section 3, to fluctuations in the presence of self-consistent Coulomb screening; the task divides neatly into a part governed by the equilibrium properties of an inhomogeneous conductor, and a part governed purely by non-equilibrium scattering. Section 4 applies the screening formalism to the proof of the fluctuation-dissipation (or Johnson–Nyquist) relation for inhomogeneous mesoscopic systems. We discuss the difference between dissipation and fluctuations in the non-linear regime. This leads into section 5, in which we describe a new class of experiments by which these differences could be measured. In section 6 we summarize.

2. Kinetic formalism

2.1. Single-particle transport

We review the structure of our kinetic model, before systematizing its self-consistent Coulomb corrections. The equation of motion for the fluctuations is derived by variational analysis of the semi-classical Boltzmann transport equation for the electron distribution function $f_\alpha(t) \equiv f_s(\boldsymbol{r}, \boldsymbol{k}, t)$. This is

$$\left(\frac{\partial}{\partial t} + D_\alpha[\boldsymbol{E}(\boldsymbol{r}, t)]\right) f_\alpha(t) = -\mathcal{W}_\alpha[f]. \tag{1}$$

The notation is as follows. Labels $\alpha = \{\boldsymbol{r}, \boldsymbol{k}, s\}$, $\alpha' = \{\boldsymbol{r}', \boldsymbol{k}', s'\}$ and so on will denote points in single-particle phase space. A sub-label s will index both the discrete sub-bands (or valleys) of a multi-level system and the spin state. The system is acted upon by the total internal field

$\boldsymbol{E}(\boldsymbol{r}, t)$, entering via the convective operator

$$D_\alpha[\boldsymbol{E}] \equiv \boldsymbol{v}_{\boldsymbol{k}} \cdot \frac{\partial}{\partial \boldsymbol{r}} - \frac{e\boldsymbol{E}}{\hbar} \cdot \frac{\partial}{\partial \boldsymbol{k}}.$$

The collision operator $\mathcal{W}_\alpha[f]$ may be for any combination of single-particle impurity scattering and two-particle scattering. Its kernel is local in real space and satisfies detailed balance. (Pauli blocking of the outgoing scattering states means that $\mathcal{W}$ is at least a quadratic functional of f.) In a system with ν dimensions, the identity operator is

$$\mathcal{I}_{\alpha\alpha'} \equiv \delta_{ss'} \left\{ \frac{\delta_{\boldsymbol{r}\boldsymbol{r}'}}{\Omega(\boldsymbol{r})} \right\} \{\Omega(\boldsymbol{r})\delta_{\boldsymbol{k}\boldsymbol{k}'}\} \longleftrightarrow \delta_{ss'}\delta(\boldsymbol{r}-\boldsymbol{r}')(2\pi)^\nu \delta(\boldsymbol{k}-\boldsymbol{k}').$$

At equilibrium, equation (1) is

$$D_\alpha[\boldsymbol{E}_0(\boldsymbol{r})] f_\alpha^{\mathrm{eq}}(t) = 0 = -\mathcal{W}_\alpha[f^{\mathrm{eq}}]. \tag{2}$$

The internal field $\boldsymbol{E}_0(\boldsymbol{r})$, defined in the absence of a driving field, satisfies

$$\frac{\partial}{\partial \boldsymbol{r}} \cdot \epsilon \boldsymbol{E}_0 = -4\pi e \Big(\langle f^{\mathrm{eq}}(\boldsymbol{r}) \rangle - n_{\mathrm{d}}(\boldsymbol{r}) \Big) \tag{3}$$

in terms of the dielectric constant $\epsilon(\boldsymbol{r})$, the electron density $\langle f^{\mathrm{eq}}(\boldsymbol{r})\rangle \equiv \Omega(\boldsymbol{r})^{-1} \sum_{\boldsymbol{k},s} f_\alpha^{\mathrm{eq}}$, and the positive background density $n_{\mathrm{d}}(\boldsymbol{r})$. The mean total particle number within the conductor is $\sum_r \Omega(\boldsymbol{r}) \langle f^{\mathrm{eq}}(\boldsymbol{r})\rangle = N$ and the equilibrium function is

$$f_\alpha^{\mathrm{eq}} = \left[1 + \exp\left(\frac{\varepsilon_\alpha - \phi_\alpha}{k_{\mathrm{B}} T} \right) \right]^{-1}. \tag{4}$$

Here the local conduction-band energy $\varepsilon_\alpha = \varepsilon_s(\boldsymbol{k}; \boldsymbol{r})$ can have band parameters that depend *implicitly* on position ($\partial/\partial \boldsymbol{r}$ does not act upon ε_α). The local Fermi level $\phi_\alpha = \mu - V_0(\boldsymbol{r})$ is the difference of the global chemical potential μ and the electrostatic potential $V_0(\boldsymbol{r})$, whose gradient is $e\boldsymbol{E}_0(\boldsymbol{r})$. Both V_0 and $\boldsymbol{E}_0$ must vanish asymptotically in the macroscopic leads carrying current into and out of the mesoscopic device.

In steady state, the non-equilibrium problem can be solved for the difference $g_\alpha = f_\alpha - f_\alpha^{\mathrm{eq}}$. After subtracting both sides of equation (2) from those of equation (1), the equation for g is

$$\sum_{\alpha'} \Big(\mathcal{I}_{\alpha\alpha'} D_{\alpha'}[\boldsymbol{E}(\boldsymbol{r}')] + \mathcal{W}^{(1)}_{\alpha\alpha'}[f] \Big) g_{\alpha'} = \frac{e\widetilde{\boldsymbol{E}}(\boldsymbol{r})}{\hbar} \cdot \frac{\partial f_\alpha^{\mathrm{eq}}}{\partial \boldsymbol{k}} - \mathcal{W}^{(2)}_\alpha[g]. \tag{5}$$

This contains the difference field $\widetilde{\boldsymbol{E}}(\boldsymbol{r}) = \boldsymbol{E}(\boldsymbol{r}) - \boldsymbol{E}_0(\boldsymbol{r})$ and two auxiliary collision operators. The operator $\mathcal{W}^{(1)}_{\alpha\alpha'}[f] \equiv \delta\mathcal{W}_\alpha[f]/\delta f_{\alpha'}$ is the linearized term. On the right-hand side we have the residual non-linear part

$$\mathcal{W}^{(2)}_\alpha[g] = \mathcal{W}_\alpha[f] - \mathcal{W}_\alpha[f^{\mathrm{eq}}] - \sum_{\alpha'} \mathcal{W}^{(1)}_{\alpha\alpha'}[f] g_{\alpha'}.$$

For elastic impurity scattering, $\mathcal{W}^{(2)} = 0$; for inelastic one-body processes, $\mathcal{W}^{(2)}$ is bilinear in g. Note that, although $\mathcal{W}[f^{\mathrm{eq}}]$ is zero, its functional derivative $\mathcal{W}^{(1)}[f^{\mathrm{eq}}]$ does not vanish identically. Hence we must carry $\mathcal{W}[f^{\mathrm{eq}}]$ formally in equation (5), since its variation is needed below.

Next, write the difference field as $\widetilde{\boldsymbol{E}}(\boldsymbol{r}) = \boldsymbol{E}_{\mathrm{ext}}(\boldsymbol{r}) + \boldsymbol{E}_{\mathrm{ind}}(\boldsymbol{r})$. The first term $\boldsymbol{E}_{\mathrm{ext}}$ is the external driving field, while $\boldsymbol{E}_{\mathrm{ind}}$ is the induced field whose Poisson equation is

$$\frac{\partial}{\partial \boldsymbol{r}} \cdot \epsilon \boldsymbol{E}_{\mathrm{ind}} = -4\pi e \Big(\langle f(\boldsymbol{r})\rangle - \langle f^{\mathrm{eq}}(\boldsymbol{r})\rangle \Big) = -4\pi e \langle g(\boldsymbol{r})\rangle. \tag{6}$$

This assumes that the background charge density $n_{\mathrm{d}}(\boldsymbol{r})$ is independent of the driving field. The non-equilibrium solution g has two properties. From equation (5) one sees that it is

generated by an inhomogeneous driving term, dependent on the equilibrium state through the factor $\partial f_{\boldsymbol{k}}^{\mathrm{eq}}/\partial \boldsymbol{k}$. Moreover, equation (6) ensures that g goes to zero with $\widetilde{\boldsymbol{E}}$, so the adiabatic connection of the steady state to the equilibrium state is maintained.

As with equation (2), the solution to equation (5) complies with two asymptotic conditions for the source and drain reservoirs: (i) *local equilibrium* and (ii) *local charge balance* (in three dimensions this means strict neutrality, while in a two-dimensional quantum-confined structure this means that the remote ionized donor layer stabilizes the confined electron gas). The active region must include the carriers in the boundary layers between the conductor and its source and drain, out to several screening lengths. Beyond this, the internal fields are negligible. For numerical convenience one shorts out the fields so that, within the operative channel geometry, $\boldsymbol{E}(\boldsymbol{r}) = \boldsymbol{E}_0(\boldsymbol{r}) = \boldsymbol{0}$ outside the boundaries of the device. Gauss's theorem implies that the active device remains globally neutral:

$$\sum_{\boldsymbol{r}} \Omega(\boldsymbol{r})\langle g(\boldsymbol{r})\rangle \equiv \sum_{\alpha} g_\alpha = 0. \tag{7}$$

Hence $\sum_\alpha f_\alpha = \sum_\alpha f_\alpha^{\mathrm{eq}} = N$. The mean total carrier number is constant.

2.2. Adiabatic fluctuations

Non-equilibrium electron–hole fluctuations in the steady state are built up through the adiabatic propagator [12]

$$G_{\alpha\alpha'} \stackrel{\mathrm{def}}{=} \frac{\delta g_\alpha}{\delta f_{\alpha'}^{\mathrm{eq}}} \tag{8}$$

with a global constraint coming directly from equation (7):

$$\sum_{\alpha} G_{\alpha\alpha'} = 0. \tag{9}$$

The equation for G follows by taking variations with respect to f^{eq} on both sides of equation (5). We obtain

$$\sum_{\beta}\Big(\mathcal{I}_{\alpha\beta} D_\beta[\boldsymbol{E}(\boldsymbol{r}_\beta)] + \mathcal{W}^{(1)}_{\alpha\beta}[f]\Big) G_{\beta\alpha'} = \mathcal{I}_{\alpha\alpha'} \frac{e\widetilde{\boldsymbol{E}}(\boldsymbol{r}')}{\hbar}\cdot\frac{\partial}{\partial \boldsymbol{k}'} - \mathcal{W}^{(1)}_{\alpha\alpha'}[f] + \mathcal{W}^{(1)}_{\alpha\alpha'}[f^{\mathrm{eq}}]. \tag{10}$$

The variation is restricted to exclude the reaction of $\boldsymbol{E}_0$ and $\boldsymbol{E}$. Thus G is a 'proper' response function, free of Coulomb screening. Here we treat the electrons as an effectively neutral Fermi liquid. Section 3 below has the complete fluctuation structure, including Coulomb effects.

One of the most important features of equation (10) is its validity for both single-particle and two-particle scattering. This can be checked by direct expansion of its right-hand collision terms, given an elastic two-body kernel. It means that the whole kinetic noise formalism is immediately applicable to semi-classical electron–electron scattering.

All of the physical consequences of the theory will therefore hold equally well when electron–electron collisions are significant. The most notable such consequence is the overall thermal-temperature scaling of non-equilibrium fluctuations for degenerate electrons.

The operator G acts upon the equilibrium Fermi-liquid fluctuations. These are electron–hole pair excitations; in our model they are given by the static long-wavelength limit of the free-electron polarization function [3], normalized by the thermal energy:

$$\Delta f_\alpha^{\mathrm{eq}} \equiv k_{\mathrm{B}} T \frac{\partial f_\alpha^{\mathrm{eq}}}{\partial \phi_\alpha} = f_\alpha^{\mathrm{eq}}(1 - f_\alpha^{\mathrm{eq}}). \tag{11}$$

Now consider the two-point electron–hole correlation $\Delta f^{(2)}_{\alpha\alpha'} \equiv (\mathcal{I}_{\alpha\alpha'} + G_{\alpha\alpha'})\,\Delta f^{\rm eq}_{\alpha'}$. The trace over α' of this elementary, non-equilibrium pair excitation is

$$\Delta f_\alpha = \sum_{\alpha'} \Delta f^{(2)}_{\alpha\alpha'} = \Delta f^{\rm eq}_\alpha + \sum_{\alpha'} G_{\alpha\alpha'}\,\Delta f^{\rm eq}_{\alpha'} \tag{12}$$

which is readily shown to be an exact solution to the linearized Boltzmann equation:

$$\sum_\beta \Big(\mathcal{I}_{\alpha\beta} D_\beta[\boldsymbol{E}(\boldsymbol{r}_\beta)] + \mathcal{W}^{(1)}_{\alpha\beta}[f]\Big)\,\Delta f_\beta = 0. \tag{13}$$

Once the explicit form of G is known, the thermal fluctuation structure of the steady state is completely specified by equation (12). The most important implication of equations (9) and (12) is conservation of the total fluctuation strength over the conductor. Thus

$$\sum_\alpha \Delta f_\alpha = \sum_\alpha \Delta f^{\rm eq}_\alpha \equiv \Delta N. \tag{14}$$

This is the direct consequence of asymptotic neutrality and equilibrium. Our next task is to show how the dynamical Boltzmann equation indeed engenders G, and hence the explicit form of the adiabatic fluctuations.

2.3. Dynamic fluctuations

The adiabatic distribution Δf is the average strength of non-equilibrium electron–hole excitations generated by spontaneous energy exchange with the thermal bath, say at time t'. The evolution of these spontaneous pair processes is governed by the inhomogeneous time-dependent Boltzmann equation

$$\sum_\beta \left\{\mathcal{I}_{\alpha\beta}\left(\frac{\partial}{\partial t} + D_\beta[\boldsymbol{E}(\boldsymbol{r}_\beta)]\right) + \mathcal{W}^{(1)}_{\alpha\beta}[f]\right\} R_{\beta\alpha'}(t-t') = \delta(t-t')\mathcal{I}_{\alpha\alpha'} \tag{15}$$

for the retarded Green function

$$R_{\alpha\alpha'}(t-t') \stackrel{\rm def}{=} \theta(t-t')\frac{\delta f_\alpha(t)}{\delta f_{\alpha'}(t')} \tag{16}$$

with initial value $R_{\alpha\alpha'}(0) = \mathcal{I}_{\alpha\alpha'}$. As with G, the variation is restricted. The double-time fluctuation

$$\Delta f^{(2)}_{\alpha\alpha'}(t-t') = R_{\alpha\alpha'}(t-t')\,\Delta f_{\alpha'} \tag{17}$$

carries information on dynamic electron–hole processes in the driven system. It is the basis for all the correlations of physical interest. Formally, $\Delta f^{(2)}_{\alpha\alpha'}(t-t')$ represents the strength of a fluctuation at point α at time t following an initial, spontaneous excitation at (α', t') whose strength is $\Delta f_{\alpha'}$.

In the frequency domain, the Fourier-transformed propagator $\mathcal{R}(\omega) = \int {\rm d}t\ {\rm e}^{{\rm i}\omega t} R(t)$ has the equation of motion

$$\sum_\beta \mathcal{B}_{\alpha\beta}(\omega)\mathcal{R}_{\beta\alpha'}(\omega) \equiv \sum_\beta \Big\{\mathcal{I}_{\alpha\beta}(D_\beta[\boldsymbol{E}(\boldsymbol{r}_\beta)] - {\rm i}\omega) + \mathcal{W}^{(1)}_{\alpha\beta}[f]\Big\}\mathcal{R}_{\beta\alpha'}(\omega) = \mathcal{I}_{\alpha\alpha'} \tag{18}$$

where $\mathcal{B}(\omega)$ is the linearized dynamical Boltzmann operator (shown in full in the middle expression), for which $\mathcal{R}(\omega)$ is the inverse. The normalization of $\mathcal{R}(\omega)$ is conserved: $\sum_\alpha \mathcal{R}_{\alpha\alpha'}(\omega) = -1/{\rm i}(\omega + {\rm i}0^+)$. This can be established from equation (18) taken together with its adjoint [13]. Comparison of equation (18) with equation (15) in its adiabatic $t \to \infty$ limit shows that the low-frequency form of $R_{\alpha\alpha'}(\omega)$ must be

$$\mathcal{R}_{\alpha\alpha'}(\omega) \to -\frac{1}{{\rm i}(\omega + {\rm i}0^+)}\frac{\Delta f_\alpha}{\Delta N}. \tag{19}$$

This asymptote retains no memory of the initial state α'.

To make the connection between the adiabatic response G in the system and the dynamic one, $\mathcal{R}$, we start from an identity for the right-hand side of equation (10):

$$\begin{aligned}\mathcal{I}_{\alpha\alpha'}\frac{e\widetilde{\boldsymbol{E}}(\boldsymbol{r}')}{\hbar}\cdot\frac{\partial}{\partial\boldsymbol{k}'} - \mathcal{W}^{(1)}_{\alpha\alpha'}[f] + \mathcal{W}^{(1)}_{\alpha\alpha'}[f^{\mathrm{eq}}]& \\ = \{\mathcal{I}_{\alpha\alpha'}(D_{\alpha'}[\boldsymbol{E}_0(\boldsymbol{r}')] - \mathrm{i}\omega) + \mathcal{W}^{(1)}_{\alpha\alpha'}[f^{\mathrm{eq}}]\}& \\ - \{\mathcal{I}_{\alpha\alpha'}(D_{\alpha'}[\boldsymbol{E}(\boldsymbol{r}')] - \mathrm{i}\omega) + \mathcal{W}^{(1)}_{\alpha\alpha'}[f]\} = \mathcal{B}^{\mathrm{eq}}_{\alpha\alpha'}(\omega) - \mathcal{B}_{\alpha\alpha'}(\omega).& \qquad (20)\end{aligned}$$

The object $\mathcal{B}^{\mathrm{eq}}(\omega)$ is the linearized equilibrium operator. We can continue equation (10) into the frequency domain, introducing the operator $\mathcal{G}(\omega)$ as the solution to

$$\mathcal{B}(\omega)\mathcal{G}(\omega) = \mathcal{B}^{\mathrm{eq}}(\omega) - \mathcal{B}(\omega) \qquad (21)$$

where we adopt an abbreviated notation for inner products. One way of expressing the solution in terms of the equilibrium resolvent $\mathcal{R}^{\mathrm{eq}}(\omega) = (\mathcal{B}^{\mathrm{eq}})^{-1}(\omega)$ is

$$\mathcal{R}(\omega) = [\mathcal{I} + \mathcal{G}(\omega)]\mathcal{R}^{\mathrm{eq}}(\omega). \qquad (22)$$

In the zero-frequency limit, one can equate residues at the pole $\omega = 0$ to get

$$\frac{\Delta f}{\Delta N} = [\mathcal{I} + \mathcal{G}(0)]\frac{\Delta f^{\mathrm{eq}}}{\Delta N}. \qquad (23)$$

This is equation (12) with G identified as $\mathcal{G}(0)$.

To determine the form of $\mathcal{G}(\omega)$, we solve equation (21) differently:

$$\mathcal{G}(\omega) = \mathcal{R}(\omega)\left(\mathcal{I}\frac{e\widetilde{\boldsymbol{E}}(\boldsymbol{r}')}{\hbar}\cdot\frac{\partial}{\partial\boldsymbol{k}'} - \mathcal{W}^{(1)}[f] + \mathcal{W}^{(1)}[f^{\mathrm{eq}}]\right). \qquad (24)$$

The resolvent $\mathcal{R}(\omega)$ can be split into its proper adiabatic part, dominant at low frequency, and a correlated part $\mathcal{C}_{\alpha\alpha'}(\omega)$ which expresses all of the dynamics:

$$\mathcal{R}_{\alpha\alpha'}(\omega) = \mathcal{C}_{\alpha\alpha'}(\omega) - \frac{1}{\mathrm{i}(\omega + \mathrm{i}0^+)}\frac{\Delta f_\alpha}{\Delta N}. \qquad (25)$$

In the frequency domain, the correlated propagator $\mathcal{C}$ satisfies the pair of sum rules [13]

$$\sum_{\alpha'}\mathcal{C}_{\alpha\alpha'}(\omega)\,\Delta f_{\alpha'} = 0 \qquad \text{for all } \alpha \qquad (26a)$$

$$\sum_{\alpha}\mathcal{C}_{\alpha\alpha'}(\omega) = 0 \qquad \text{for all } \alpha'. \qquad (26b)$$

With these we can resume the calculation of $\mathcal{G}(\omega)$. The adiabatic part of $\mathcal{R}(\omega)$ makes no contribution to the right-hand sum in equation (24); it decouples both from the trace $\langle\partial F/\partial\boldsymbol{k}'\rangle'$, which vanishes over the space of physical distributions $F_{\alpha'}$, and from $\sum_\beta \mathcal{W}^{(1)}_{\beta\alpha'}$, which is identically zero. Since $\mathcal{C}(\omega \to 0)$ is regular and well defined, we finally have

$$G_{\alpha\alpha'} = \mathcal{G}_{\alpha\alpha'}(0) = \sum_\beta \mathcal{C}_{\alpha\beta}(0)\left(\mathcal{I}_{\beta\alpha'}\frac{e\widetilde{\boldsymbol{E}}(\boldsymbol{r}')}{\hbar}\cdot\frac{\partial}{\partial\boldsymbol{k}'} - \mathcal{W}^{(1)}_{\beta\alpha'}[f] + \mathcal{W}^{(1)}_{\beta\alpha'}[f^{\mathrm{eq}}]\right). \qquad (27)$$

This result completes the steady-state description. Alongside equation (22) it shows the intimate connection between adiabatic fluctuations, via $G = \mathcal{G}(0)$, and transient ones, via $\mathcal{C}$. The steady-state fluctuation structure is one with the dynamics.

3. Coulomb correlations

So far we have revisited three cardinal ideas:

(a) the governing role of asymptotic neutrality and equilibrium;

(b) the form of the non-equilibrium electron–hole fluctuations as linear functionals of the equilibrium ones; and

(c) the analytic connection between dynamics and adiabaticity.

We now apply these principles to mesoscopic conductors as Coulomb systems, to obtain the following properties:

(1) renormalization of equilibrium fluctuations by long-range screening;
(2) short-ranged collisional screening away from equilibrium; and

(3) the full form of the current–current correlation function.

3.1. Equilibrium screening

We start by calculating Coulomb screening within the conductor at equilibrium. In a *correlated* system, the total fluctuation is

$$\widetilde{\Delta} f_\alpha^{\rm eq} = k_{\rm B}T\,\frac{\delta f_\alpha^{\rm eq}}{\delta\mu} = \frac{\delta\phi_\alpha}{\delta\mu}\,\Delta f_\alpha^{\rm eq} \tag{28}$$

where we have used equation (4) for $f_\alpha^{\rm eq}$. Freezing the response of the internal field means that $\delta\phi_\alpha/\delta\mu = 1$. Inclusion of the self-consistent response means that

$$\frac{\delta\phi_\alpha}{\delta\mu} = 1 - \frac{\delta V_0(\boldsymbol{r})}{\delta\mu} = 1 - \sum_{\boldsymbol{r}'}\Omega(\boldsymbol{r}')V_{\rm C}(\boldsymbol{r}-\boldsymbol{r}')\,\frac{\delta}{\delta\mu}\Big(\langle f^{\rm eq}(\boldsymbol{r}')\rangle - n_{\rm d}(\boldsymbol{r}')\Big) \tag{29}$$

where $V_{\rm C}(\boldsymbol{r}) = e^2/\epsilon|\boldsymbol{r}|$ is the Coulomb interaction. Equation (29) follows from the integral form of equation (3).

We must address the response of the neutralizing background $n_{\rm d}(\boldsymbol{r})$. Asymptotically, the source and drain regions are *unconditionally* neutral. This requires that $n_{\rm d}(\boldsymbol{r}) \equiv \langle f^{\rm eq}(\boldsymbol{r})\rangle$ always and everywhere outside the conductor [14]. Beyond the sample boundaries, $V_{\rm C}$ is completely screened out. Thus the contribution of the integrand on the right-hand side of equation (29) extends over the active region Ω.

Within the conductor as such, the behaviour of $n_{\rm d}$ involves physical processes outside the kinetic description. If the background distribution is taken as independent of transport, it is sensitive only to the equilibrium carrier distribution within Ω. The corresponding contribution to equation (29) is

$$\frac{\delta n_{\rm d}(\boldsymbol{r}')}{\delta f_{\alpha''}^{\rm eq}}\,\frac{\widetilde{\Delta} f_{\alpha''}^{\rm eq}}{k_{\rm B}T}.$$

This lets us recast equation (29) as

$$\frac{\delta\phi_\alpha}{\delta\mu} \equiv 1 - \sum_{\alpha''}\widetilde{V}_{\rm C}(\boldsymbol{r},\boldsymbol{r}'')\frac{\widetilde{\Delta} f_{\alpha''}^{\rm eq}}{k_{\rm B}T} \tag{30}$$

in terms of the effective interaction

$$\widetilde{V}_{\rm C}(\boldsymbol{r},\boldsymbol{r}'') = V_{\rm C}(\boldsymbol{r}-\boldsymbol{r}'') - \sum_{\boldsymbol{r}'}\Omega(\boldsymbol{r}')V_{\rm C}(\boldsymbol{r}-\boldsymbol{r}')\frac{\delta n_{\rm d}(\boldsymbol{r}')}{\delta f_{\alpha''}^{\rm eq}}. \tag{31}$$

In the most common case, the background $n_{\rm d}$ does not change (the rigid-jellium model), so there is no compensatory feedback from background screening: $\widetilde{V}_{\rm C} = V_{\rm C}$. In the opposite extreme we have $\delta n_{\rm d}(\boldsymbol{r}')/\delta f^{\rm eq}_{\alpha''} = \delta_{\boldsymbol{r}'\boldsymbol{r}''}/\,\Omega(\boldsymbol{r}')$, leading to complete cancellation of the local field generated by the carrier fluctuations. We will assume *at most* an overall dependence on N. That is, $\delta n_{\rm d}/\delta f^{\rm eq}_{\alpha'} = \delta n_{\rm d}/\delta N$ for all α'. This applies to cases such as the modulation-doped heterojunction [11], an important example of a structure whose background donors and active carriers are coupled via N, but with incomplete compensation owing to the physical separation of carriers and donors.

A closed form for $\widetilde{\Delta} f^{\rm eq}$ can be obtained by combining equations (28) and (30). Rather than follow that course, it is more revealing to analyse the problem kinetically.

3.2. Boltzmann formulation of screening

The kinetic equation for $\widetilde{\Delta} f^{\rm eq}$ is obtained by operating on equation (2) for the equilibrium distribution. We have

$$\sum_{\alpha'} \mathcal{B}^{\rm eq}_{\alpha\alpha'}(0)\,\widetilde{\Delta} f^{\rm eq}_{\alpha'} - \left(\frac{e}{\hbar}\sum_{\alpha'}\frac{\delta \boldsymbol{E}_0(\boldsymbol{r})}{\delta f^{\rm eq}_{\alpha'}}\,\widetilde{\Delta} f^{\rm eq}_{\alpha'}\right)\cdot\frac{\partial f^{\rm eq}_{\alpha}}{\partial \boldsymbol{k}} = 0. \tag{32}$$

The second term on the left-hand side is the response of the self-consistent field. This equation is a reformulation of equation (28); the distribution $\widetilde{\Delta} f^{\rm eq}$ given by the latter satisfies equation (32) when detailed balance is used to eliminate the collisional contribution, giving $\mathcal{B}^{\rm eq}(0)\,\widetilde{\Delta} f^{\rm eq} = D[\boldsymbol{E}_0]\,\widetilde{\Delta} f^{\rm eq}$. However, the specifically kinetic structure of equation (32) provides a rather different window on the screening physics.

Equation (3) implies that, within Ω, the variation of $e\boldsymbol{E}_0$ with respect to $f^{\rm eq}$ is the effective electrostatic force due to an electron:

$$e\frac{\delta \boldsymbol{E}_0(\boldsymbol{r})}{\delta f^{\rm eq}_{\alpha'}} \equiv -e\widetilde{\boldsymbol{E}}_{\rm C}(\boldsymbol{r},\boldsymbol{r}') = \frac{\partial}{\partial \boldsymbol{r}}\widetilde{V}_{\rm C}(\boldsymbol{r},\boldsymbol{r}'). \tag{33}$$

Consequently equation (32) becomes

$$\sum_{\alpha'} \mathcal{B}^{\rm eq}_{\alpha\alpha'}(0)\,\widetilde{\Delta} f^{\rm eq}_{\alpha'} = -\frac{e}{\hbar}\frac{\partial f^{\rm eq}_{\alpha}}{\partial \boldsymbol{k}}\cdot\sum_{\alpha'}\widetilde{\boldsymbol{E}}_{\rm C}(\boldsymbol{r},\boldsymbol{r}')\,\widetilde{\Delta} f^{\rm eq}_{\alpha'}. \tag{32'}$$

As a variant of the equilibrium Boltzmann equation, equation (32′) is inhomogeneous. Its general solution includes a term proportional to the homogeneous solution, the proper fluctuation $\Delta f^{\rm eq}$. Let $\gamma_{\rm C}$ be the proportionality constant. Then

$$\widetilde{\Delta} f^{\rm eq}_{\alpha} = \gamma_{\rm C}\,\Delta f^{\rm eq}_{\alpha} - \frac{e}{\hbar}\sum_{\beta}\mathcal{C}^{\rm eq}_{\alpha\beta}(0)\frac{\partial f^{\rm eq}_{\beta}}{\partial \boldsymbol{k}_{\beta}}\cdot\sum_{\alpha'}\widetilde{\boldsymbol{E}}_{\rm C}(\boldsymbol{r}_{\beta},\boldsymbol{r}')\,\widetilde{\Delta} f^{\rm eq}_{\alpha'} \tag{34}$$

where $\mathcal{C}^{\rm eq}(\omega)$ is the correlated part of the equilibrium resolvent $\mathcal{R}^{\rm eq}(\omega)$. The integral on the right-hand side of equation (34) has a structure similar to equation (27), in that the adiabatic part of $\mathcal{R}^{\rm eq}$ makes no contribution after decoupling of the intermediate wave-vector sums.

With equation (26*b*), which gives $\sum_{\alpha}\mathcal{C}^{\rm eq}_{\alpha\alpha'} = 0$, summation over α in equations (34) and (28) produces

$$\gamma_{\rm C} = \frac{\widetilde{\Delta} N}{\Delta N} = \sum_{\alpha}\frac{\delta\phi_{\alpha}}{\delta\mu}\frac{\Delta f^{\rm eq}_{\alpha}}{\Delta N} \tag{35}$$

where $\widetilde{\Delta} N \equiv \sum_{\alpha}\widetilde{\Delta} f^{\rm eq}_{\alpha}$. In figure 1 we plot $\gamma_{\rm C}$ for the two-dimensional (2D) electron gas forming the quantum-confined metallic channel in a typical AlGaAs/InGaAs/GaAs heterojunction [11]. The strong, self-consistent Coulomb contribution to the energy levels in the

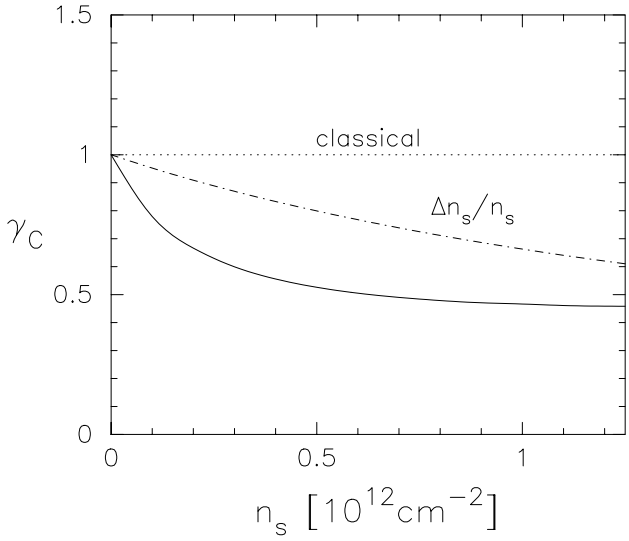

Figure 1. Self-consistent Coulomb screening of a two-dimensional electron gas, confined within a typical III–V heterojunction quantum well at $T = 300$ K. Solid line: the suppression factor γ_C for screening-induced reduction of the electron–hole fluctuation density, below its free value. At high density, carriers are sharply localized in the direction transverse to current flow. Fluctuations in their electrostatic energy provide negative feedback to reduce the free fluctuations in density (see the appendix). Dash–dotted line: ratio of the free electron–hole fluctuation density to the carrier density; the more degenerate the carriers, the smaller the ratio. Dotted line: the classical limit for both γ_C and $\Delta n_s/n_s$.

quantum well gives a pronounced dependence on carrier density. In turn, this produces the negative feedback suppressing the analogue to $\delta\phi_\alpha/\delta\mu$ within this system. The theory of such systems is reviewed in the appendix.

The resulting γ_C has immediate practical relevance to heterojunction device engineering. This factor modifies not only the random thermal fluctuations, but also the systematic variations of carrier distribution in response to a gate potential [11]; these determine the channel's current gain and differential capacitance (see equation (A.6)). Through its direct effect on such quantities, γ_C is a significant aspect of high-performance transistor design.

We can compare equation (35) with the variation in the *contact potential* for transferring a conduction electron from reservoir to sample. The mean electrostatic potential per carrier is

$$u_{\rm cp} = \frac{1}{N}\sum_{\boldsymbol{r}} \Omega(\boldsymbol{r}) V_0(\boldsymbol{r}) \langle f^{\rm eq}(\boldsymbol{r})\rangle = \frac{1}{N}\sum_{\alpha} (\mu - \phi_\alpha) f_\alpha^{\rm eq}. \tag{36}$$

Its variation is

$$\frac{\delta u_{\rm cp}}{\delta\mu} = \frac{1}{N}\sum_{\alpha}\left[\left(1 - \frac{\delta\phi_\alpha}{\delta\mu}\right) f_\alpha^{\rm eq} + (\mu - \phi_\alpha - u_{\rm cp})\frac{\widetilde{\Delta} f_\alpha^{\rm eq}}{k_{\rm B}T}\right]$$

which can be brought into the form

$$\frac{\delta u_{\rm cp}}{\delta\mu} = 1 - \gamma_C + \sum_{\boldsymbol{r}} \Omega(\boldsymbol{r}) \frac{\delta V_0(\boldsymbol{r})}{\delta\mu}\left(\frac{\langle f^{\rm eq}(\boldsymbol{r})\rangle}{N} - \frac{\langle \Delta f^{\rm eq}(\boldsymbol{r})\rangle}{\Delta N}\right)$$
$$+ \frac{1}{k_{\rm B}T}\sum_{\boldsymbol{r}} \Omega(\boldsymbol{r})\Big(V_0(\boldsymbol{r}) - u_{\rm cp}\Big)\langle\widetilde{\Delta} f^{\rm eq}(\boldsymbol{r})\rangle. \tag{37}$$

Evidently there is a close relationship between the contact potential and γ_C, acting to renormalize the equilibrium thermal fluctuations in an inhomogeneous sample. For typical problems involving piecewise-uniform metallic structures, the correction terms in equation (37) will be small, so $\gamma_C \to 1 - \delta u_{cp}/\delta\mu$.

Figure 2 recapitulates the thermodynamic origin of u_{cp} in a strongly metallic sample, placed between less-metallic leads. This is a type of p–n junction formation. Figure 3 contrasts the effect of complete versus partial compensation of carrier fluctuations by the background response. In figure 3(a) the response of the neutralizing background in the sample totally compensates the carrier fluctuations (as one assumes asymptotically in the leads). The contact potential is immune to the presence of Δf^{eq}, which retains its bare, free-Fermi-liquid form. In figure 3(b), the response of the background fails to screen out completely the Coulomb

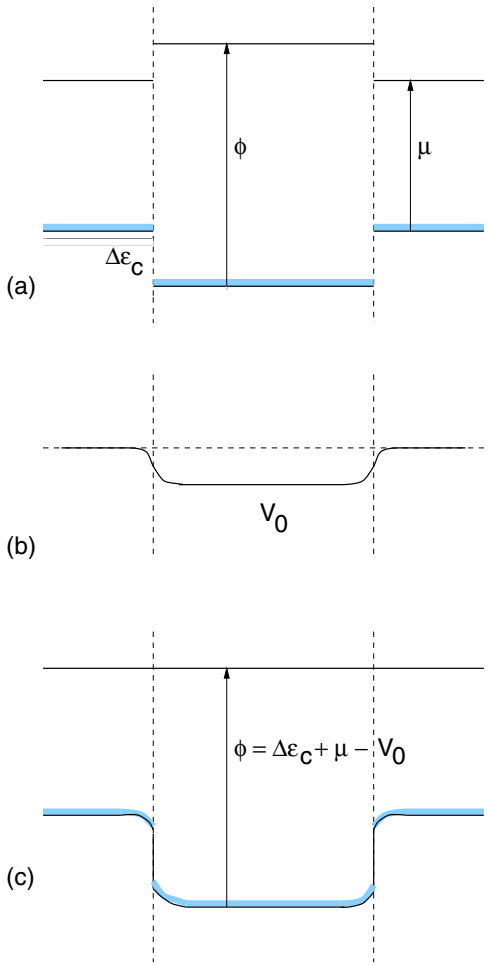

Figure 2. Schematic realignment of the metallic band in a conductor terminated by dissimilar source and drain contacts. (a) Bare conduction bands. (b) An electrostatic contact potential V_0 develops at the interfaces. This is generated by the net transfer of electrons, as in p–n junctions. (c) The complete non-uniform band structure: V_0 depends on the global chemical potential μ. The rigid conduction-band offset $\Delta\varepsilon_c$ does not. For dense carrier populations, most of the non-uniformity is concentrated at the interfaces.

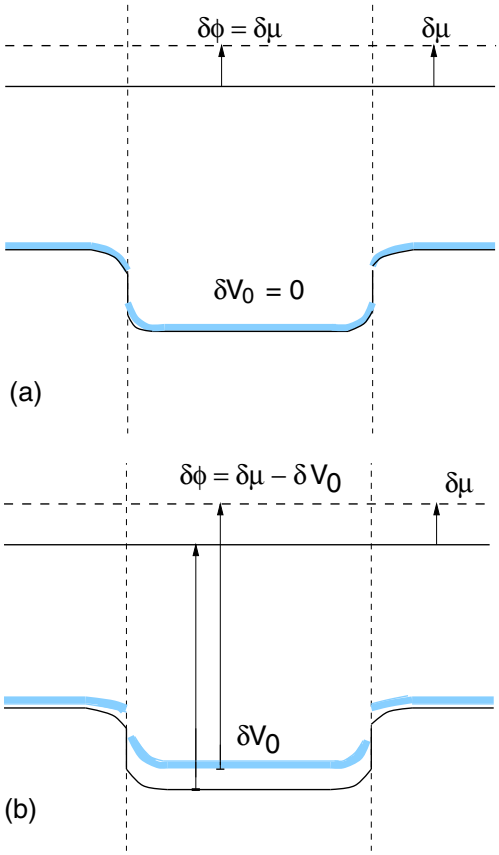

Figure 3. The response of the local electrochemical potential ϕ in a non-uniform conductor–lead geometry. (a) Complete compensation of carrier fluctuations by the positive background. A change in global chemical potential μ produces no change in the local electrostatic potential V_0. The net change in the internal field, due to free-carrier fluctuations, is negated by a matching fluctuation in background density. The free Fermi-liquid fluctuations are unchanged. (b) Incomplete compensation. A global change $\delta\mu$ induces a local change δV_0 in the electrostatic potential, partly offsetting $\delta\mu$. Now the free Fermi-liquid fluctuations are *suppressed* by interfacial Coulomb screening.

potential generated by $\Delta f^{\rm eq}$. The negative self-consistent feedback on the fluctuations leads to their suppression.

The Boltzmann equation (34) is solved by introducing an equilibrium Coulomb screening operator $\Gamma^{\rm eq}(\omega)$, whose inverse is

$$\{(\Gamma^{\rm eq})^{-1}\}_{\alpha\alpha'}(\omega) = \mathcal{I}_{\alpha\alpha'} + \frac{e}{\hbar}\sum_{\beta} \mathcal{C}^{\rm eq}_{\alpha\beta}(\omega)\frac{\partial f^{\rm eq}_{\beta}}{\partial \boldsymbol{k}_{\beta}} \cdot \widetilde{\boldsymbol{E}}_{\rm C}(\boldsymbol{r}_{\beta}, \boldsymbol{r}') \tag{38a}$$

leading to the closed form

$$\widetilde{\Delta} f^{\rm eq}_{\alpha} = \gamma_{\rm C} \sum_{\alpha'} \Gamma^{\rm eq}_{\alpha\alpha'}(0)\, \Delta f^{\rm eq}_{\alpha'}. \tag{38b}$$

In a non-uniform mesoscopic system, this is the counterpart to the random-phase approximation for the static, screened fluctuations in the standard rigid-jellium model of the uniform electron gas [3].

3.3. Collision-mediated screening

We now look beyond equilibrium, to the influence of dynamical scattering. We start by defining the Coulomb screening operator in an external driving field, through the inverse

$$\{\Gamma^{-1}\}_{\alpha\alpha'}(\omega) \stackrel{\text{def}}{=} \mathcal{I}_{\alpha\alpha'} + \frac{e}{\hbar}\sum_{\beta}\mathcal{C}_{\alpha\beta}(\omega)\frac{\partial f_\beta}{\partial \boldsymbol{k}_\beta}\cdot\widetilde{\boldsymbol{E}}_{\mathrm{C}}(\boldsymbol{r}_\beta,\boldsymbol{r}'). \tag{39}$$

The central role of this operator will become clear shortly. Here we note that, while γ_{C} is collisionless, Γ exhibits the interplay of scattering and screening, which is an exclusively non-equilibrium process. Equation (26*b*) means that Γ has the property

$$\sum_{\alpha}\Gamma_{\alpha\alpha'}(\omega) = 1 \qquad \text{for all } \alpha'. \tag{40}$$

This states that, while Γ can shift fluctuation strength at shorter scales within the structure, unlike γ_{C} it cannot redistribute fluctuation strength globally throughout the whole conductor.

As a basic illustration of collisional screening, in figure 4 we plot, for a simplified model [2], the collisional suppression factor $\gamma_{\mathrm{coll}}(q)$. This is related to the real-space Fourier transform of Γ via

$$\gamma_{\mathrm{coll}}(q) = \frac{2}{\Omega}\sum_{\boldsymbol{k}}\Gamma_{\boldsymbol{k}\boldsymbol{k}'}(\boldsymbol{q},\omega=0)$$

which turns out to be independent of the second wave vector $\boldsymbol{k}'$. This partial average retains enough structure to be a useful guide to the physics of Γ. For technical details, see reference [2]. At long wavelengths, $\gamma_{\mathrm{coll}}(q) \to 1$. There is no suppression because the collisional corrections are primarily local; the effective interaction is fully screened by the leads in the asymptotic

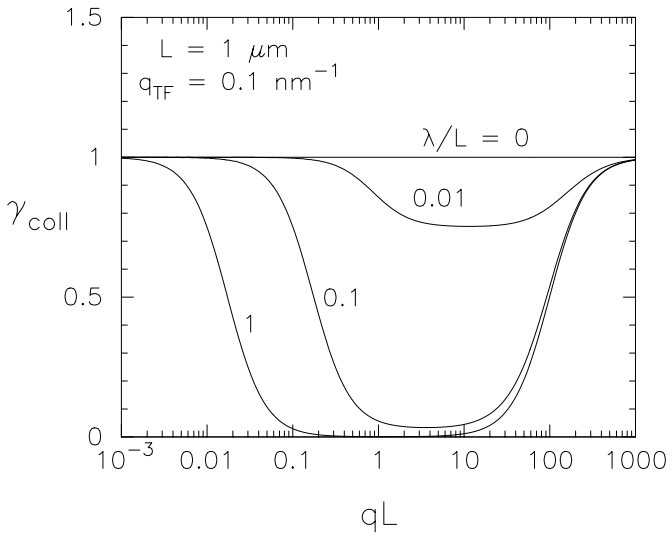

Figure 4. The Fourier transform of the collision-mediated screening factor in the low-field limit of the inelastic Drude model for degenerately doped GaAs, at density 10^{18} cm^{-3}. The parameters are the Thomas–Fermi wave vector q_{TF}, the mean free path λ, and the sample length L. For $q > q_{\mathrm{TF}}$ (short range) the Coulomb interaction is numerically small; there is no screening of free electron–hole fluctuations. For $q_{\mathrm{TF}} > q > (q_{\mathrm{TF}}L\lambda)^{-1}$ (moderate range) the Coulomb interaction is stronger, leading to appreciable collisional suppression. For $q \ll L^{-1}$ (long range) the Coulomb interaction is fully screened by the macroscopic leads. Again there is no suppression. Collisional screening, unlike contact-potential screening, cannot renormalize fluctuations macroscopically.

limit, and accounts for the inability of Γ to renormalize fluctuations across the structure. At very short wavelengths, all screening (even full Thomas–Fermi) is irrelevant simply because the bare interaction itself vanishes as $q^{-2} \ll 1$. Again there is no suppression and $\gamma_{\rm coll}(q) \to 1$.

Collisional suppression is most effective at intermediate distances. There it reflects the interplay of material and geometrical parameters. One can expect a rich variety of behaviours in this regard.

The formal significance of the non-equilibrium screening operator is that it links the screening-free propagators $\mathcal{R}$ and $\mathcal{C}$ to their Coulomb-screened forms, $\widetilde{\mathcal{R}}$ and $\widetilde{\mathcal{C}}$. In shortened notation, the linearized Boltzmann equation with screening is

$$\mathcal{B}(\omega)\widetilde{\mathcal{R}}(\omega) = \mathcal{I} - \left(\frac{e}{\hbar}\frac{\partial f}{\partial \boldsymbol{k}} \cdot \widetilde{\boldsymbol{E}}_{\rm C}\right)\widetilde{\mathcal{R}}(\omega). \tag{41}$$

The same field $\widetilde{\boldsymbol{E}}_{\rm C}$ as is defined in equation (33) appears in both equilibrium and non-equilibrium situations. That is because equations (3) and (6), combined, lead to $\delta \boldsymbol{E}(\boldsymbol{r})/\delta f_{\alpha'} = -\widetilde{\boldsymbol{E}}_{\rm C}(\boldsymbol{r}, \boldsymbol{r}')$. The latter result holds under the assumption that $n_{\rm d}(\boldsymbol{r})$ depends at most on N, whose constancy away from equilibrium is enforced by the overall neutrality of the conductor through equation (7).

Similarly, $\widetilde{\Delta} N$ is also invariant. This follows from the zero norm of the total adiabatic propagator $\widetilde{G} \equiv \delta g/\delta f^{\rm eq}$, when summed over the active volume. We see that $\widetilde{G}$ is now the screened version of the proper variational derivative G. Exactly as with G, it is the global neutrality of g (equation (7) again) that guarantees the sum rule for the steady-state fluctuation $\widetilde{\Delta} f$:

$$\sum_{\alpha} \widetilde{\Delta} f_{\alpha} = \sum_{\alpha} \left(\widetilde{\Delta} f_{\alpha}^{\rm eq} + \sum_{\beta} \widetilde{G}_{\alpha\beta}\, \widetilde{\Delta} f_{\beta}^{\rm eq}\right) = \widetilde{\Delta} N.$$

This is the formal analogue to the screening-free equation (14). We do not elaborate the detailed form of $\widetilde{G}$, except to note that it is the zero-frequency limit of the following screened operator (compare equations (21) and (22)):

$$\widetilde{\mathcal{G}}(\omega) = \widetilde{\mathcal{R}}(\omega)(\widetilde{\mathcal{R}}^{\rm eq})^{-1}(\omega) - \mathcal{I}.$$

Equation (41) can be solved with $\mathcal{R}$:

$$\widetilde{\mathcal{R}}(\omega) = \mathcal{R}(\omega) - \mathcal{C}(\omega)\left(\frac{e}{\hbar}\frac{\partial f}{\partial \boldsymbol{k}} \cdot \widetilde{\boldsymbol{E}}_{\rm C}\right)\widetilde{\mathcal{R}}(\omega) = \Gamma(\omega)\mathcal{R}(\omega). \tag{42}$$

In the screening term of the middle expression, we omit the non-contributing adiabatic part of $\mathcal{R}$, since $\mathcal{C}$ alone operates non-trivially within the integral. In the rightmost expression, we have integrated the screening term by applying equation (39). A match of residues in equation (42), at the pole $\omega = 0$, provides the result (compare equation (23))

$$\widetilde{\Delta} f_{\alpha} = \widetilde{\Delta} N \sum_{\alpha'} \Gamma_{\alpha\alpha'}(0)\frac{\Delta f_{\alpha'}}{\Delta N} = \gamma_{\rm C} \sum_{\alpha'} \Gamma_{\alpha\alpha'}(0)\, \Delta f_{\alpha'}. \tag{43}$$

This is the first of two key equations of this section. It affords an exact and explicit formula for the steady-state fluctuations, extending the random-phase screened equation (38*b*) to the driven system. Above all, it establishes that the non-equilibrium, Coulomb-screened thermal fluctuations *must scale linearly with the equilibrium renormalization* $\gamma_{\rm C}$. This, in turn, means that the current auto-correlation will also be proportional to $\gamma_{\rm C}$. We emphasize that this scaling principle is the strict outcome of asymptotic equilibrium and charge balance in the macroscopic leads.

3.4. Dynamics and the current auto-correlation

We now obtain the dynamics with screening, given by the correlated part $\widetilde{\mathcal{C}}$ of the resolvent $\widetilde{\mathcal{R}}$. Equation (42) is equivalent to

$$\widetilde{\mathcal{C}}(\omega) - \frac{\mathcal{I}}{\mathrm{i}(\omega + \mathrm{i}0^+)} \frac{\widetilde{\Delta} f}{\widetilde{\Delta} N} = \Gamma(\omega) \left(\mathcal{C}(\omega) - \frac{\mathcal{I}}{\mathrm{i}(\omega + \mathrm{i}0^+)} \frac{\Delta f}{\Delta N} \right) \tag{44}$$

which can be rearranged, first with help from equation (43), then by taking the product with $\widetilde{\Delta} f$ on both sides and invoking the screened form of equation (26*a*), $\widetilde{\mathcal{C}}(\omega)\, \widetilde{\Delta} f = 0$:

$$\widetilde{\mathcal{C}}(\omega) = \Gamma(\omega)\mathcal{C}(\omega) - \frac{\Gamma(\omega) - \Gamma(0)}{\mathrm{i}(\omega + \mathrm{i}0^+)} \frac{\Delta f}{\Delta N} = \Gamma(\omega)\mathcal{C}(\omega) \left(\mathcal{I} - \frac{\widetilde{\Delta} f}{\widetilde{\Delta} N} \right). \tag{45}$$

This is the second key equation of the model. An alternative form, for later use, is

$$\Gamma(\omega)\mathcal{C}(\omega) = \widetilde{\mathcal{C}}(\omega) \left(\mathcal{I} - \frac{\Delta f}{\Delta N} \right). \tag{46}$$

With equation (45) we gain the last component of the microscopic velocity auto-correlation, fully screened. In complete analogy with the screening-free correlation [1], its zero-frequency form is

$$\langle\langle \boldsymbol{v} \boldsymbol{v}' \, \widetilde{\Delta} \mathrm{f}^{(2)}(\boldsymbol{r}, \boldsymbol{r}'; 0) \rangle\rangle_{\mathrm{c}}' \stackrel{\mathrm{def}}{=} \frac{1}{\Omega(\boldsymbol{r})} \frac{1}{\Omega(\boldsymbol{r}')} \sum_{\boldsymbol{k},s} \sum_{\boldsymbol{k}',s'} \boldsymbol{v}_{\boldsymbol{k}s} \widetilde{\mathcal{C}}_{\alpha\alpha'}(0) \boldsymbol{v}_{\boldsymbol{k}'s'} \, \widetilde{\Delta} f_{\alpha'} \tag{47}$$

where we keep the correlated part of $\widetilde{\Delta} \mathrm{f}^{(2)}_{\alpha\alpha'}(\omega) = \widetilde{\mathcal{R}}_{\alpha\alpha'}(\omega)\, \widetilde{\Delta} f_{\alpha'}$. At finite frequency one must add the displacement-current contribution to the fluctuations. The velocity is replaced with the non-local operator

$$\boldsymbol{u}_{\boldsymbol{k}s}(\boldsymbol{r}, \boldsymbol{r}''; \omega) \equiv \frac{\delta_{ss''}\delta_{\boldsymbol{r}\boldsymbol{r}''}}{\Omega(\boldsymbol{r}'')} \boldsymbol{v}_{\boldsymbol{k}''s''} - \frac{\mathrm{i}\omega\epsilon}{4\pi e} \boldsymbol{E}_{\mathrm{C}}(\boldsymbol{r} - \boldsymbol{r}'') \tag{48}$$

where $e\boldsymbol{E}_{\mathrm{C}}(\boldsymbol{r} - \boldsymbol{r}'') \equiv -\partial V_{\mathrm{C}}(\boldsymbol{r} - \boldsymbol{r}'')/\partial \boldsymbol{r}$ is the bare Coulomb force, coupling to the dynamically changing carrier density. The introduction of displacement currents means that two additional, intermediate sums over region Ω must be incorporated within the expectation $\langle\langle \mathrm{Re}\{\boldsymbol{u}\, \widetilde{\Delta} \mathrm{f}^{(2)}\, \boldsymbol{u}'^{*}\} \rangle\rangle_{\mathrm{c}}'$. At $\omega = 0$ it reverts to equation (47).

As with the screening-free version [1], equation (47) conveys the physics of the spontaneous electron–hole excitation $\widetilde{\Delta} \mathrm{f}^{(2)}$ in the metallic conduction band. Its average thermal strength is $\widetilde{\Delta} f$, giving the initial flux contribution $\boldsymbol{v}'\, \widetilde{\Delta} f$. Once it is spontaneously excited out of the steady-state background, the thermal pair excitation evolves semi-classically *in keeping with the full Boltzmann equation of motion*, equation (41), characterized by $\widetilde{\mathcal{C}}(\omega)$. The final weighting by $\boldsymbol{v}$ sets up the auto-correlation of the microscopic flux.

The whole process describes the non-equilibrium development of the electron–hole fluctuations, starting out as elementary excitations out of the steady state of the electron liquid and relaxing dynamically back to the steady state. There is much more in equation (47) than in its unscreened analogue [1].

4. The fluctuation-dissipation theorem

The fluctuation-dissipation theorem (FDT) is the prime relation of linear low-field transport, and its derivation within a given model gives first-hand evidence of that model's internal consistency. We prove that the FDT is satisfied within our description of non-uniform mesoscopic Coulomb systems; since our approach is a kinetic-theoretical one, our derivation of the mesoscopic FDT is closer to van Kampen's programme for response theory [16] than

it is to Kubo's programme [17]. The former viewpoint is better suited to non-perturbative calculations in the high-field regime, which is important for applications to practical devices where one must go beyond the FDT. After establishing the theorem, we look at its experimental implications.

To make contact with measurable properties such as the conductance and the thermal noise-power spectrum, both the single-particle distribution g and the two-particle correlation $\langle\langle \boldsymbol{v}\boldsymbol{v}' \, \Delta \mathrm{f}^{(2)}(0)\rangle\rangle'_{\mathrm{c}}$ should be related directly to the externally applied field $\boldsymbol{E}_{\mathrm{ext}}$. (For convenience we take a uniform dielectric constant ϵ.) Recalling the Poisson equation (6) for the induced field $\boldsymbol{E}_{\mathrm{ind}}$ as a functional of g, the kinetic equation (5) for g itself can be transformed:

$$\begin{aligned}\mathcal{B}(0)g &= \frac{e}{\hbar}(\boldsymbol{E}_{\mathrm{ext}} + \boldsymbol{E}_{\mathrm{ind}}[g]) \cdot \frac{\partial f^{\mathrm{eq}}}{\partial \boldsymbol{k}} - \mathcal{W}^{(2)}[g] \\ &= -\frac{e}{k_{\mathrm{B}}T}\boldsymbol{E}_{\mathrm{ext}} \cdot \boldsymbol{v}\, \Delta f^{\mathrm{eq}} - \frac{e}{\hbar}\frac{\partial f^{\mathrm{eq}}}{\partial \boldsymbol{k}} \cdot (\boldsymbol{E}_{\mathrm{C}} g) - \mathcal{W}^{(2)}[g]. \end{aligned} \tag{49}$$

In the leading term on the right-hand side of the second line, the identity $\partial f^{\mathrm{eq}}/\partial \boldsymbol{k} = -(\hbar \boldsymbol{v}_{\boldsymbol{k}}/k_{\mathrm{B}}T)\Delta f^{\mathrm{eq}}$ has been used†. In the middle right-hand term, the bare Coulomb force arises from the integral form of the Poisson equation for $\boldsymbol{E}_{\mathrm{ind}}[g]$.

We now make the first of two simplifying assumptions: let the neutralizing background of the conductor stay fixed (rigid jellium), so that it does not cancel the self-consistent response of the fluctuations. Then the effective field $\widetilde{\boldsymbol{E}}_{\mathrm{C}}$ becomes identical with $\boldsymbol{E}_{\mathrm{C}}$, and we may integrate the Boltzmann equation (49) by invoking the screening operator Γ:

$$\begin{aligned} g &= -\mathcal{C}(0)\left(\frac{e}{k_{\mathrm{B}}T}\boldsymbol{E}_{\mathrm{ext}} \cdot \boldsymbol{v}\, \Delta f^{\mathrm{eq}} + \frac{e}{\hbar}\frac{\partial (f-g)}{\partial \boldsymbol{k}} \cdot (\boldsymbol{E}_{\mathrm{C}} g) + \mathcal{W}^{(2)}[g]\right) \\ &\equiv -\frac{e}{k_{\mathrm{B}}T}\Gamma(0)\mathcal{C}(0)(\boldsymbol{E}_{\mathrm{ext}} \cdot \boldsymbol{v}\, \Delta f^{\mathrm{eq}}) + h \end{aligned} \tag{50}$$

where

$$h = \Gamma(0)\mathcal{C}(0)\left(\frac{e}{\hbar}\frac{\partial g}{\partial \boldsymbol{k}} \cdot (\boldsymbol{E}_{\mathrm{C}} g) - \mathcal{W}^{(2)}[g]\right)$$

is the remnant non-linear term, of order g^2. Next we construct the total resistive power dissipation $P(\Omega)$ over the conducting region Ω. The local current density is $\boldsymbol{J}(\boldsymbol{r}) = -e\langle \boldsymbol{v} g(\boldsymbol{r})\rangle$, so

$$P(\Omega) = \sum_{\boldsymbol{r}} \Omega(\boldsymbol{r})\widetilde{\boldsymbol{E}}(\boldsymbol{r}) \cdot \boldsymbol{J}(\boldsymbol{r}) = -e\sum_{\alpha}(\boldsymbol{E}_{\mathrm{ext}} \cdot \boldsymbol{v})_{\alpha} g_{\alpha}. \tag{51}$$

There is no contribution to P from the induced field $\boldsymbol{E}_{\mathrm{ind}}(\boldsymbol{r})$. First, the total current I is conserved since the flux density, integrated over any directed surface bisecting the conductor, is constant. Second, the internal Coulomb forces are conservative so the difference in *induced* potential between two points on any circuit path through Ω, one point deep in the source and the other in the drain, must vanish. Thus the current may be factored out of the right-hand side of equation (51) to leave just a line integral for the total electromotive potential V and resulting in the canonical dissipation formula $P(\Omega) = IV$. The microscopic proof of this fundamental transport theorem, under the most general mesoscopic conditions, was given recently by Magnus and Schoenmaker [18]; it is absolutely essential to the FDT.

Note that $P(\Omega)$ is finite and calculable if and only if Ω is indeed a *bounded* region. Once again the hypotheses of equilibrium and charge balance in the leads are indispensable.

† The term $\boldsymbol{E} \cdot \partial f^{\mathrm{eq}}/\partial \boldsymbol{k}$ can be traced back to a commutator of current and density operators in the underlying quantum-dynamical description. As the semi-classical limit of a current–density response function, its association with the electron–hole correlation $-\boldsymbol{v}\, \partial f^{\mathrm{eq}}/\partial \mu$ is not fortuitous.

Our other simplifying assumption is that the applied field $\boldsymbol{E}_{\rm ext}$ is uniform over Ω, with no internal sources. We take the field to be constant along the x-axis, whose sense is from source to the drain†. Let it be $-V/L$ over sample length L. Equations (50) and (51) combined lead to

$$P(\Omega) = \sum_{\alpha}(-e\boldsymbol{E}_{\rm ext}\cdot\boldsymbol{v})_{\alpha}\left(\frac{1}{k_{\rm B}T}\sum_{\alpha'}[\Gamma(0)\mathcal{C}(0)]_{\alpha\alpha'}(-e\boldsymbol{E}_{\rm ext}\cdot\boldsymbol{v})_{\alpha'}\,\Delta f^{\rm eq}_{\alpha'} + h_{\alpha}\right)$$
$$\equiv \frac{1}{k_{\rm B}T}\left(\frac{eV}{L}\right)^2\sum_{\alpha\alpha'} v_x[\Gamma(0)\mathcal{C}(0)]_{\alpha\alpha'}v'_x\,\Delta f^{\rm eq}_{\alpha'} + P_h(\Omega). \tag{52}$$

The contribution $P_h(\Omega)$ is of order Vg^2; this non-linear complement is negligible in the weak-field limit. The first line of equation (52) highlights the fact that the mean dissipative power is itself an *auto-correlation function for the power density* $\boldsymbol{E}_{\rm ext}\cdot\boldsymbol{J}$. This is the heart of the FDT, in the context of electron transport.

To make a final connection with device parameters, we express the current phenomenologically as the usual relation $I \equiv GV$, where G is the conductance (not to be confused with earlier notation for the adiabatic propagator). Since $P = IV$, we have

$$G = P(\Omega)/V^2 \equiv \mathcal{S}(0)/4k_{\rm B}T \tag{53}$$

in terms of the low-frequency current-noise spectral density [15, 19]

$$\mathcal{S}(0) = 4\sum_{\alpha\alpha'}(-ev_x/L)[\Gamma(0)\mathcal{C}(0)]_{\alpha\alpha'}(-ev'_x/L)\,\Delta f^{\rm eq}_{\alpha'} + \mathcal{S}_h(0) \tag{54}$$

where the non-linear correction $\mathcal{S}_h(0) = 4k_{\rm B}T\,P_h(\Omega)/V^2$ is well defined; close to equilibrium, only the first right-hand sum survives. Note, however, that equation (54) holds for arbitrary driving fields.

Equations (53) and (54) recover the Johnson–Nyquist formula and establish the fluctuation-dissipation theorem for a mesoscopic system, complete with Coulomb effects. The theorem relates the empirical Joule-heating rate over the sample, determined by G, to the equilibrium noise power determined by $\mathcal{S}(0)$. We remark that *collisional screening*, through $\Gamma(0)$ in equation (54), will be an important modifier of the conductance in short non-uniform samples.

We must now address the role of the *microscopic* current auto-correlation, equation (47). Let

$$\widetilde{\mathcal{S}}(0) = 4\sum_{\boldsymbol{r}}\Omega(\boldsymbol{r})\sum_{\boldsymbol{r}'}\Omega(\boldsymbol{r}')\langle\langle(-ev_x/L)(-ev'_x/L)\,\widetilde{\Delta}\mathrm{f}^{(2)}(\boldsymbol{r},\boldsymbol{r}';0)\rangle\rangle'_{\rm c}$$
$$= 4\frac{e^2}{L^2}\sum_{\alpha\alpha'} v_x\widetilde{\mathcal{C}}(0)_{\alpha\alpha'}v'_x(\widetilde{\Delta}f^{\rm eq} + \widetilde{\mathcal{G}}(0)\,\widetilde{\Delta}f^{\rm eq})_{\alpha'}$$
$$\equiv 4\frac{e^2}{L^2}\sum_{\alpha\alpha'} v_x\widetilde{\mathcal{C}}(0)_{\alpha\alpha'}v'_x\,\widetilde{\Delta}f^{\rm eq}_{\alpha'} + \widetilde{\mathcal{S}}_g(0). \tag{55}$$

This fully non-equilibrium spectral density includes the well-defined excess contribution $\widetilde{\mathcal{S}}_g(0) \propto \widetilde{\mathcal{G}}(0)\,\widetilde{\Delta}f^{\rm eq}$; in the weak-field limit we may drop this, since it vanishes with g. With equation (46) we transform (54) into

$$\mathcal{S}(0) = 4\frac{e^2}{L^2}\sum_{\alpha\alpha'} v_x\left[\widetilde{\mathcal{C}}(0)\left(\mathcal{I} - \frac{\Delta f}{\Delta N}\right)\right]_{\alpha\alpha'}v'_x\,\Delta f^{\rm eq}_{\alpha'} = 4\frac{e^2}{L^2}\sum_{\alpha\alpha'} v_x\widetilde{\mathcal{C}}(0)_{\alpha\alpha'}v'_x\,\Delta f^{\rm eq}_{\alpha'}. \tag{56}$$

† This discussion is easily widened to other device geometries in a source-free driving field, by solving the transport problem in a curvilinear coordinate frame which (a) preserves the local cell volume $\Omega(\boldsymbol{r})$ and (b) has its local $\boldsymbol{x}$-axis proportional to $-\boldsymbol{E}_{\rm ext}/E^2_{\rm ext}$.

The sum for $(v_x\widetilde{\mathcal{C}}(0)\,\Delta f)(v_x'\,\Delta f^{\rm eq})$ vanishes because $\langle v_x\widetilde{\mathcal{C}}(0)\,\Delta f\rangle$ decouples from $\langle v_x'\,\Delta f^{\rm eq}\rangle' = 0$. By adding and subtracting $\widetilde{\Delta} f^{\rm eq}_{\alpha'}/\gamma_{\rm C}$ from $\Delta f^{\rm eq}_{\alpha'}$ on the right-hand side of equation (56) we obtain the strictly low-field relation

$$\mathcal{S}(0) = \frac{1}{\gamma_{\rm C}}\left(\widetilde{\mathcal{S}}(0) - 4\frac{e^2}{L^2}\sum_{\alpha\alpha'} v_x\widetilde{\mathcal{C}}(0)_{\alpha\alpha'}v_x'(\widetilde{\Delta} f^{\rm eq}_{\alpha'} - \gamma_{\rm C}\,\Delta f^{\rm eq}_{\alpha'})\right). \tag{57a}$$

The link between $\mathcal{S}(0)$ and $\widetilde{\mathcal{S}}(0)$ is most transparent when the correction term is small. This will be so under the same conditions in which the relation $\gamma_{\rm C} \approx 1 - \delta u_{\rm cp}/\delta\mu$ applies: the non-uniformity of the structure must reside mostly in the interfacial zones, which will be short compared to L so they do not overlap. Then $\widetilde{\Delta} f^{\rm eq} \to \gamma_{\rm C}\,\Delta f^{\rm eq}$ and

$$\mathcal{S}(0) \to \widetilde{\mathcal{S}}(0)/\gamma_{\rm C}. \tag{57b}$$

Before discussing the experimental consequences of this section, we make an important observation. In comparing the noise spectral densities of equations (54) and (55), the crucial point is not the striking difference in scale, $\gamma_{\rm C}$, but the fact that *they evolve as completely different mathematical structures*. In the high-field limit one or both of the purely non-linear contributions, $\mathcal{S}_h(0)$ and $\widetilde{\mathcal{S}}_g(0)$, will not be negligible.

Even when the sample and its reservoirs are of identical material, with no contact-potential effects, the correlation functions which generate $\mathcal{S}(0)$ and $\widetilde{\mathcal{S}}(0)$ do *not* describe the same underlying physics. The former deals with resistive dissipation of electrical energy, while the latter deals with the dynamical spreading of correlations in real space and momentum space. These are quite distinct (though clearly connected) dynamical processes. Their separateness should manifest itself in the non-equilibrium current noise, and nowhere more strongly than in a *Coulomb-correlated system*.

5. Experimental implications

5.1. Equilibrium versus high fields

The fluctuation-dissipation theorem asserts the necessary equivalence of $\mathcal{S}(0)$ and $\widetilde{\mathcal{S}}(0)$ in the weak-field limit (leaving aside, for now, the contact potential); clearly, the FDT can say nothing about the non-equilibrium region. In GD we discussed the main physical distinction between resistive and non-linear fluctuations [1, 2]: the existence of *non-dissipative* excess noise entering via $\widetilde{\mathcal{S}}_g(0)$.

Our model demands that, for degenerate conductors, hot-electron noise must remain proportional to ambient temperature. Here we recall the indirect but strong evidence in favour of overall T-scaling of thermal noise, in the observations of Liefrink *et al* for 2D mesoscopic wires [20]. They anticipated—yet failed to detect—a large supra-thermal signature from electron–electron scattering (two-body collisional effects are particularly enhanced in 2D). According to standard estimates, that signature *should* have scaled with a hot-carrier temperature $T_{\rm e} \gg T$. Remarkably enough, no such contribution was seen.

The experimental reality and accessibility of non-equilibrium, non-dissipative fluctuations is not at issue. Shot noise, proportional to the current, is the foremost example of a stochastic process not associated with dissipation (else equation (53) would entail a linear current-dependent correction in G; such a correction is not observed with shot noise). As far as noise that is generated strictly thermally is concerned, there are two major experimental questions, as yet unanswered:

- Where is the onset of hot-electron behaviour in mesoscopic noise?
- What is its temperature dependence in the degenerate regime?

Even without heavy calculation [2, 21] our formalism predicts the growth of hot-electron noise, scaling with T. At the same time, theoretical work by Shimizu and Ueda [22] and by Liu, Eastman and Yamamoto [23] suggests that the T-independent shot noise should be attenuated as inelastic phonon scattering begins to outweigh elastic impurity scattering at higher fields.

5.2. Screening

We come to the physics of contact-potential screening. Our equation (55) unambiguously requires suppression of the intrinsic current fluctuations for an inhomogeneous sample, relative to the resistive Johnson–Nyquist spectral density for the same system. We emphasize that the correlation $\widetilde{\mathcal{S}}(0)$ follows directly from the exact solution to the linearized, fully self-consistent, Boltzmann equation. Since $\widetilde{\mathcal{S}}(0)$ dictates the fluctuations underlying the system, it is in no sense an artificial construct; it conveys crucial physical information.

At low fields, the factoring-out of γ_{C} in equation (57) 'saves the phenomenon' of the FDT for the power density by removing all trace of contact-potential effects from the microscopic spectral density, which then recovers the conventional Johnson–Nyquist form†. An identical rescaling procedure, preserving the FDT, has been discussed by Kittel [24] for a dissipative system in which (non-dissipative) active feedback works to reduce the fluctuations.

At high fields, the story takes a very different turn. The specific functional form of $\widetilde{\mathcal{S}}_g(0)$ means that one should see suppression at least in the *hot-electron noise* [1]. This should survive any of the calibration protocols commonly used to separate the sought-after noise signal from the unwanted background.

In the quasi-equilibrium limit, the choice of calibration strategy is crucial to the acquisition of data on screening in noise. For example, the equilibrium term was subtracted from the early 2D measurements of Liefrink *et al* for shot noise [20]. Screening-suppressed current noise, if present in the quiescent state, would have been lost from their low-field signal. In other cases, such as the Steinbach *et al* shot-noise experiment on three-dimensional (3D) silver wires [25], the equilibrium noise floor is used as an absolute check on the measurement scale. However, since $\gamma_C = 1$ in homogeneous 3D metallic structures, there is no screening to detect. Therefore, a combination of 2D wire samples (for screening) and non-subtractive calibration (to keep the equilibrium current correlations) would be ideal.

5.3. Experiments suggested

Testing the suppression theory calls for a specific structure in which screening is not only present, but easily controlled. Such an arrangement was suggested in reference [9]. Here we show it in figure 5, with more detail. It relies on a back-gated diffusive wire, fabricated on a heterojunction quantum-well substrate to provide the conductive medium: a uniform, two-dimensional and tunable metallic electron gas. The theory of γ_{C} in quantum-confined 2D electron gases is reviewed in the appendix (see also figure 1). Although γ_{C} in 2D differs mathematically from its 3D form (section 2.2 above), their Coulomb origins and physical consequences are the same.

In a channel that is uniform in the plane of conduction, the non-linear term $\mathcal{S}_h(0)$ of equation (54) contains no screening correction. It depends only on the residual collision

† It is worth remarking that $\gamma_{\mathrm{C}} \to 1$ always in a classical system; equipartition of energy guarantees that Maxwellian fluctuations are not suppressed in this fashion. It is only for strongly degenerate, interacting particles that equipartition fails (the Fermi–Dirac distribution is not separable into purely kinetic and purely potential factors). The screened current fluctuations are related to fluctuations of kinetic energy, representing only part of the total thermal energy $k_{\mathrm{B}}T$ per excitable mode in the system; the rest is bound up in the Coulomb complement.

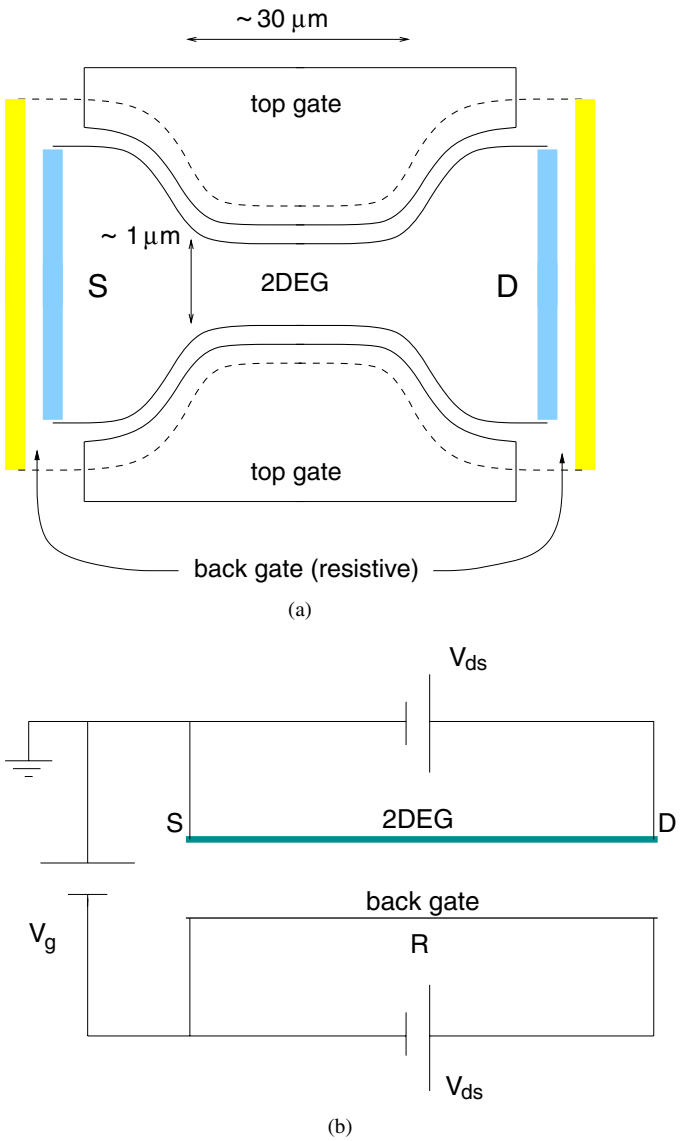

Figure 5. A schematic diagram of a III–V heterojunction-based, two-dimensional electron gas (2DEG) for observing the Coulomb suppression of current noise. (a) Plan view. The 2DEG wire is connected to large 2DEG source and drain leads. The wire is defined by a negatively biased, split *top gate*. Uniform control of the density throughout wire and leads is achieved by a bias voltage applied to a *back gate*, beneath the channel. The back gate should be wider than the wire to minimize edge effects, but narrow enough to form a resistive strip of the same length as the wire. (b) Side view, with the arrangement of back-bias and source–drain voltages. To minimize non-uniformities in the channel's density profile at higher currents, the resistive back gate should support its own driving voltage, to match the source–drain voltage across the channel. This offsets the local fall-off in channel potential and keeps the effective bias constant along the wire.

kernel $\mathcal{W}^{(2)}$ (within h in equation (50)), which is zero for pure elastic scattering (although inelastic phonon and inter-band scattering will revive it for large enough V). Homogeneity also means that any correction terms, such as the one on the right-hand side of equation (57),

are small†. Thus, of the two disparate non-linear corrections $\mathcal{S}_h(0)$ and $\widetilde{\mathcal{S}}_g(0)$, only the latter is significant and should appear explicitly as part of the thermal current–current correlation signal. Its characteristic experimental signatures should be

(a) linear dependence of excess noise on bath temperature T, and
(b) linear dependence of excess noise on the suppression factor γ_C.

Any controlled modulation of T and γ_C should be mirrored in the non-equilibrium noise spectrum. Previously we have reasoned [2, 9] that (b), just like (a), cannot apply to shot noise; its contribution to the total dissipationless noise should depend neither on T nor on γ_C. This means that thermal and shot noise ought to be clearly separable from each other, according to their different response to a gate-bias potential. In this way each becomes individually accessible for study within the same experiment. A method for the direct resolution of the two kinds of noise has not been available up to now.

Such predictions, if correct, should shed light on the behaviour of current fluctuations beyond linear response. Ultimately, non-linear processes will be the determinants of device performance in technological applications of mesoscopic electronics. Any technique that advanced the development of non-equilibrium physics would be of value.

6. Summary

Our aim has been to set up a systematic account of current fluctuations in inhomogeneous metallic conductors, at mesoscopic scales. Its vehicle is the semi-classical kinetic equation, supplemented with the theory of charged Fermi liquids. These are the definitive, universally recognized tools of transport analysis.

Our conclusion is that Coulomb screening exerts a marked influence on the fluctuations of degenerate carriers in conductors that are highly non-uniform. The resulting screening-induced suppression of current noise should be observable in several different ways. An appropriate system for such measurements is the two-dimensional electron gas.

This paper's complexity makes it useful to retrace our path.

- We started with the Boltzmann description of non-equilibrium transport and fluctuations for a mesoscopic conductor, connected to macroscopic current leads. The boundary conditions in the leads, namely *local neutrality* and *local equilibrium* at all times, govern the form of the non-equilibrium fluctuations within the driven conductor. The absolutely cardinal role of these constraints cannot be stressed enough. In particular they ensure that the thermal noise of a degenerate system, even away from equilibrium, always retains its characteristic equilibrium feature: proportionality to the ambient temperature.
- Next, we extended the kinetic description to cover the internal fields' self-consistent response to electron–hole pair excitations, which make up the fluctuations. We showed how inhomogeneity between the conductor and its electron reservoirs sets the scene for overall suppression of thermal current noise. This is produced by negative feedback from fluctuations of the contact potential between sample and reservoirs. A separate, non-equilibrium Coulomb process arises in association with scattering. This type of screening acts at shorter range, and has direct influence on the conductance if the channel is small

† One could not expect a wire, with uniform 2D density at equilibrium, to remain uniform at higher fields. In principle the spatial variation of g and Δf could be computed, with effort. More practically, it may be easier to stabilize the local density by aligning a resistive back gate to the 2D wire (figure 5(b)). A drain–source potential along the gate, mirroring V_{ds} applied along the channel, should partly offset the local effective bias in the channel, $V(x) \sim V_g + xV_{ds}/L$, which would otherwise spoil the uniform density profile.

enough. Collisional screening, unlike contact-potential screening, cannot alter the *overall* scale of noise throughout the channel.

- We then proved the fluctuation-dissipation theorem within our kinetic theory of inhomogeneous mesoscopic conductors. We pointed out the sharp physical distinction between correlations in the *power density*, which are fundamentally tied to resistive dissipation, and spontaneous fluctuations in the *current density*, which are explicitly suppressed by contact-potential screening and which develop a non-dissipative excess component out of equilibrium. Nevertheless, at low fields there exists a simple quantitative relationship between power-density and current-density fluctuations. This is required by the thermodynamics of the fluctuation-dissipation theorem.
- Last, we proposed an experimental arrangement to probe both Coulomb-screening and non-dissipative features of current noise. This is best done in a two-dimensional channel, fabricated on a back-gated heterojunction for uniform control over carrier density and the amount of screening. Care should be taken with low-field calibration of current noise in the two-dimensional wire, to retain correlations in the quiescent state.

We have aimed to keep faith with basic principles of kinetic and electron-gas theory, albeit at a price in immediate formulaic appeal. One's concern should be primarily for physical substance, less for cosmetics. Still, our own philosophy may continue to be viewed, by some, as mesoscopic revisionism in the face of a prevailing semi-classical doctrine [6]. In reality, the complete opposite is true. The present description sticks manifestly to the letter of the kinetic orthodoxy. It rejects all generic assumptions that are logically unnecessary to transport and demonstrably inconsistent with the physics of charged Fermi liquids [8, 9]. Its outcomes are entirely conservative in spirit and guaranteed to be fully conserving in practice. Our experimental predictions are specific and verifiable.

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Appendix. Screening in self-confined quantum channels

For convenience, we describe the origin of screening suppression in heterojunction quantum-well channels. Our treatment presupposes the practical solution of the bound-state Schrödinger problem in these structures. Details are in the literature [11, 26].

Assume that the bound-state energy levels $\{\varepsilon_i(n_s)\}_i$ have been computed for an ensemble of carriers, confined in the quantum well with uniform 2D density n_s. Quantization is transverse to the plane of free motion for the carriers. The conducting states form discrete sub-bands whose energy thresholds are $\varepsilon_i(n_s)$. Since the Hamiltonian includes the large self-consistent potential from mutual Coulomb repulsion of the localized electrons, the levels naturally depend on n_s. The associated equilibrium distribution in sub-band i, according to equation (4), is

$$f^{\rm eq}_{i,\boldsymbol{k}} = \left[1 + \exp\left(\frac{\varepsilon_{\boldsymbol{k}} + \varepsilon_i(n_s) - \mu}{k_{\rm B}T}\right)\right]^{-1}. \tag{A.1}$$

The density in device area Ω fixes μ implicitly through

$$n_s = \frac{2}{\Omega}\sum_{i,\boldsymbol{k}} f^{\rm eq}_{i,\boldsymbol{k}} = n^*_s \sum_i \ln\{1 + \exp[(\mu - \varepsilon_i(n_s))/k_{\rm B}T]\} \tag{A.2}$$

where $n_s^* \equiv m^* k_B T/\pi\hbar^2$ is the natural 2D density scale (m^* is the effective mass). The density of *free* electron–hole pair fluctuations is straightforward to calculate:

$$\Delta n_s = k_B T \frac{\partial n_s}{\partial \mu} = n_s^* \sum_i f_{i,\mathbf{0}}^{\mathrm{eq}}. \tag{A.3}$$

The dimensionless degeneracy factor $\Delta n_s/n_s$ is shown in figure 1. The smaller it is, the greater the degeneracy of the carriers; the closer to unity, the more classically they behave.

Equation (A.3) ignores the self-consistency of the bound-state energies. When this is included one obtains the density of *Coulomb-correlated* pair fluctuations

$$\widetilde{\Delta} n_s = k_B T \frac{\delta n_s}{\delta \mu} = n_s^* \sum_i \left(1 - \frac{\delta \varepsilon_i}{\delta \mu}\right) f_{i,\mathbf{0}}^{\mathrm{eq}}. \tag{A.4}$$

Similarly, the fluctuation in $f_{i,\boldsymbol{k}}^{\mathrm{eq}}$ is

$$\widetilde{\Delta} f_{i,\boldsymbol{k}}^{\mathrm{eq}} = \left(1 - \frac{\delta \varepsilon_i}{\delta \mu}\right) \Delta f_{i,\boldsymbol{k}}^{\mathrm{eq}}.$$

The factor in parentheses plays the role of $\delta\phi_\alpha/\delta\mu$ in our description of 3D mesoscopic screening. We have a system that is uniform in the plane of conduction but highly inhomogeneous in the transverse direction. Its reduced symmetry, which is not immediately apparent at the level of one-body transport, is nevertheless a powerful modifier of the fluctuations within the plane.

Equation (A.4) can be closed for $\widetilde{\Delta} n_s$ by writing

$$\frac{\delta \varepsilon_i}{\delta \mu} = \frac{\mathrm{d}\varepsilon_i}{\mathrm{d}n_s} \frac{\widetilde{\Delta} n_s}{k_B T}$$

so that a term proportional to $\widetilde{\Delta} n_s$ can be transferred from right to left in the equation. The result is

$$\widetilde{\Delta} n_s = n_s^* \sum\nolimits_i f_{i,\mathbf{0}}^{\mathrm{eq}} \Big/ \left(1 + \frac{n_s^*}{k_B T} \sum\nolimits_i \frac{\mathrm{d}\varepsilon_i}{\mathrm{d}n_s} f_{i,\mathbf{0}}^{\mathrm{eq}}\right) \equiv \gamma_C \, \Delta n_s. \tag{A.5}$$

This is the 2D screening-suppression factor. With allowance for dimensionality, all of the Coulomb screening theory in sections 2 and 3 of the main text is rigorously applicable to the 2D case. In practice, most carriers in a degenerate 2D channel occupy the ground-state sub-band, $i = 0$. In that case we have $\gamma_C \approx 1 - \delta\varepsilon_0/\delta\mu$, and $\widetilde{\Delta} f_{0,\boldsymbol{k}}^{\mathrm{eq}} \approx \gamma_C \, \Delta f_{0,\boldsymbol{k}}^{\mathrm{eq}}$.

Finally, we show that the same suppression factor enters directly into the measurable parameters of an active, gated structure built on the heterojunction [11]. We consider the differential capacitance of the carrier population under a biasing gate (of area Ω). A change in bias potential V_g at the gate, which overlies the modulation-doped layer above the channel, alters the profile $n_d(z)$ of the ionized donors in the transverse z-direction. In turn this alters n_s. Solution of the Poisson donor-depletion problem provides the variational derivative $\delta\mu/\delta V_g$. The channel's contribution to C_{gs}, the differential capacitance, is the change in total charge $Q = e\Omega n_s$ in the channel. Thus

$$C_{gs} \sim -\frac{\delta Q}{\delta V_g} = e\Omega \frac{\delta n_s}{\delta \mu} \left| \frac{\delta \mu}{\delta V_g} \right| = \gamma_C \left(e\Omega \frac{\partial n_s}{\partial \mu} \left| \frac{\delta \mu}{\delta V_g} \right| \right). \tag{A.6}$$

This makes it clear that suppression is an important physical phenomenon when the 2D electron gas is perturbed systematically. Onsager's regression principle [27] immediately suggests that screening must equally affect the stochastic perturbations (noise) of the self-same system. It would be very unusual if the thermal fluctuations were to bear no trace of γ_C, even as it strongly modifies the channel's differential response.

References

[1] Green F and Das M P 2000 *J. Phys.: Condens. Matter* **12** 5233

[2] Green F and Das M P 1998 *Preprint* cond-mat/9809339 (CSIRO report RPP3911, unpublished)

[3] Pines D and Nozières P 1966 *The Theory of Quantum Liquids* (New York: Benjamin)

[4] Nagaev K E 1998 *Phys. Rev.* B **57** 4628
(Nagaev K E 1998 *Preprint* cond-mat/9908193)

[5] Naveh Y, Averin D V and Likharev K K 1997 *Phys. Rev. Lett.* **79** 3482
Naveh Y, Averin D V and Likharev K K 1999 *Phys. Rev.* B **59** 2848
Naveh Y 1999 *Preprint* cond-mat/9910256

[6] Kogan Sh M 1996 *Electronic Noise and Fluctuations in Solids* (Cambridge: Cambridge University Press)

[7] Datta S 1995 *Electronic Transport in Mesoscopic Systems* (Cambridge: Cambridge University Press)

[8] Das M P and Green F 1999 *Proc. 23rd Int. Workshop on Condensed Matter Theories* ed G S Anagnostatos (New York: Nova Science) at press
(Das M P and Green F 1999 *Preprint* cond-mat/9910183)

[9] Green F and Das M P 2000 *Proc. UPoN'99: 2nd Int. Conf. on Unsolved Problems of Noise and Fluctuations (AIP Proc. 511)* ed D Abbott and L B Kish (New York: American Institute of Physics) pp 422–33
and for a similar discussion see
Green F and Das M P 1999 *Preprint* cond-mat/9905086

[10] González T, González C, Mateos J, Pardo D, Reggiani L, Bulashenko O M and Rubí J M 1998 *Phys. Rev. Lett.* **80** 2901
González T, Mateos J, Pardo D, Bulashenko O M and Reggiani L 1999 *Phys. Rev.* B **60** 2670

[11] Weisbuch C and Vinter B 1991 *Quantum Semiconductor Structures: Fundamentals and Applications* (San Diego, CA: Academic)

[12] Green F 1996 *Phys. Rev.* B **54** 4394

[13] Kogan Sh M and Shul'man A Ya 1969 *Zh. Eksp. Teor. Fiz.* **56** 862 (Engl. Transl. 1969 *Sov. Phys.–JETP* **29** 467)

[14] A change $\delta\mu$ in the chemical potential of the open leads is conceptually equivalent to replacing them with leads having the same conduction-band structure, occupied by electrons with a compensating background at the new density $n(\mu+\delta\mu)$. This constraint on neutrality in the open system contrasts with strictly closed models of the electron gas, whose total background charge never changes. There, global neutrality is automatic but transport can be simulated only through the usual Born–von Kármán periodicity. Strictly closed systems with periodic boundary conditions are commonly regarded as inadequate for non-equilibrium modelling of mesoscopic circuits. This is not necessarily so; see reference [18] below. Two other perspectives on open boundary conditions are in
Frensley W R 1990 *Rev. Mod. Phys.* **62** 745
Sols F 1991 *Phys. Rev. Lett.* **67** 2874

[15] Gantsevich S V, Gurevich V L and Katilius R 1979 *Nuovo Cimento* **2** 1

[16] van Kampen N G 1971 *Phys. Norveg.* **5** 279

[17] Kubo R, Toda M and Hashitsume N 1991 *Statistical Physics II: Nonequilibrium Statistical Mechanics* 2nd edn (Berlin: Springer)

[18] Magnus W and Schoenmaker W 1998 *J. Math. Phys.* **39** 6715

[19] Nougier J P 1994 *IEEE Trans. Electron Devices* **41** 2034

[20] Liefrink F, Dijkhuis J I, de Jong M J M, Molenkamp L W and van Houten H 1994 *Phys. Rev.* B **49** 14 066

[21] Green F and Das M P 1998 *Recent Progress in Many-Body Theories* ed D Neilson and R F Bishop (Singapore: World Scientific) p 102
(Green F and Das M P 1997 *Preprint* cond-mat/9709142)

[22] Shimizu A and Ueda M 1992 *Phys. Rev. Lett.* **69** 1403

[23] Liu R C, Eastman P and Yamamoto Y 1997 *Solid State Commun.* **102** 785

[24] Kittel C 1958 *Elementary Statistical Mechanics* (New York: Wiley) pp 152, 153
whose analysis is based on theoretical and experimental work on electrometer damping by Milatz and colleagues:
Milatz J M W and van Zolingen J J 1953 *Physica* **19** 181
Milatz J M W, van Zolingen J J and van Iperen B B 1953 *Physica* **19** 195

[25] Steinbach A H, Martinis J M and Devoret M H 1996 *Phys. Rev. Lett.* **76** 3806

[26] Bastard G 1990 *Wave Mechanics Applied to Semiconductor Structures* (Les Ulis: Les Editions de Physique) ch V

[27] van Kampen N G 1981 *Stochastic Processes in Physics and Chemistry* (Amsterdam: North-Holland) p 270